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# Understanding Geometric Pattern and its Geometry Part 7 – What can go wrong?

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## Abstract

*In the first paper[1] of this series, we discussed selected geometric concepts related to the class of patterns referred to as gereh[2]. The creation of these patterns followed the rules of gereh (axioms). Constructions of such patterns follow three steps: construction of the contour for the pattern template, construction of tessellation inside the contour, and designing a pattern for each tessellation tile.*
*In this part, we will discuss why some patterns are considered incorrect and what are the fundamental features of a geometric pattern that make it acceptable?*

## Introduction

Geometric patterns often confuse their admirers. Many of them look very simple. Just a few segments form a star or a rosette. Thus many people believe that drawing a geometric pattern is a very straightforward task. They start fiddling with segments and circles, and as a result of this activity, they get something that looks like a pattern they have seen somewhere. It looks like, but a good observer may notice many differences and inaccuracies. In their pattern drawing, people often forget about the geometry and geometric properties of a pattern. This paper will show a few of such patterns and explain what is wrong with them.

There is also another case. Some people consider themselves pattern gurus, and they often criticize other people's works, including patterns made by famous designers from the deep past. In this paper, we will show such patterns and explain the possible mistakes made by these gurus.

Finally, this paper will discuss a few controversial designs, and we will show how these patterns were constructed.

## Problems with terminology

Authors in the West often had problems with the terminology of the eastern things and concepts. The famous book by Edward W. Said (see [8] ) discussed this phenomenon in relation to culture and politics. Here we will look only at terms related to the geometric patterns. Let us look at names used by some authors. We have:

- Islamic art, geometric patterns in Islamic art (d'Avennes, 1827),
- Les Éléments de l'Art Arabe (Bourgoin, 1879),
- Arabic geometrical pattern (Bourgoin, 1973),
- Geometric patterns in Saracenic art (Hankin, 1925),
- Geometric ornament in Central Asia (Balkanov, 1940),

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[1] Understanding Geometric Pattern and its Geometry (part 1), eJMT, vol. 14, Nr 2, pages 87-106.
[2] Gerehs, or gereh patterns, this is only one group of geometric patterns. There are many other patterns that can be created using different approach, e.g. patterns on square grids, triangular or other types of grids.

- Герих (Gierih), (Balkanov, 1947),
- Гирих (Girih), (Rempel, 1961),
- Gereh (Encyclopedia Iranica, 2001),
- Islamic geometric pattern (Broug, Bonner, …).

As we can notice, some of these terms are wrong or confusing. The problem is that these authors tried to squeeze all geometric patterns under one umbrella and have a global term for them. This cannot work for many reasons.

First, let us notice that Arabs were not creators of geometric patterns. Geometric patterns came from Persia and Central Asia. Persians are the ones who created a very rich class of patterns and methods for drawing them. But no one mentions Persians.

The term 'Saracen' is quite vague. Various sources attach to this word many different meanings. So, which Saracens were involved in geometric patterns creation?

What about 'Islamic geometric patterns? This term was invented in the West and is completely wrong. While talking with Uzbek or Iranian people, I never heard anybody mentioning this term. They strongly object to this term. They say, "this is our traditional art, and it has nothing to do with Islam." In fact, geometric patterns were known in Persia before Islam came to this region (see [2] ). Iranians use the term 'gereh.'

The most proper are terms used by Russian scientists. They use the terms 'girih' or 'gierieh,' which are Central Asian versions of the Persian term 'gereh.'

We observe that in the West, people often believe that there is religious symbolism in the term 'Islamic geometric patterns,' and they try to associate with them some symbolic meaning.

There is still missing geometric meaning in any of the examples mentioned above. What is a geometric pattern from a geometric point of view? How do we classify them? From a geometric point of view, it could be convenient to use the forms of symmetry existing in these patterns. For example, a decagonal geometric pattern means a pattern with decagonal local symmetries or shapes derived from the geometry of a regular decagon. The group of decagonal geometric patterns is still huge. But at least we get a more concrete and still vague description. What about geometric patterns using local symmetries 5, 10, and 20? Are they still decagonal? Why decagonal but not pentagonal? Probably the safest could be notation listing all local symmetries of a pattern, e.g. [5,10,20]. However, none of the above names or definitions will satisfy a mathematician. They do not give us the precision required in a mathematical theory.

## Correct or incorrect pattern?

Let us consider what we mean by a correct geometric pattern? Do we have rules allowing us to decide when a geometric pattern is correct and when it is not? One of the Turkish kundekari makers once said, "*a good pattern must not hurt your eye and be doable*." What does this mean?

The first argument is a reference to our sense of aesthetics. We like some things, and we do not like others. We used to deal with some forms, and we have never seen others. We like regular shapes with at least one symmetry line. We like lines crossing smoothly without unnecessary or strange bending. Many old patterns follow our taste. But, there are still old patterns with unusual shapes and strange line bending. Are they correct or incorrect?

We will understand the second argument when we look at a work of woodworking or ceramic tiling artisans. The material they use creates some limitations on pattern design. Patterns with unusual shapes can be big trouble for a ceramic tiling craftsman. For him, it is much easier to deal with a limited number of uniform shapes in his work. He can prepare a template for each shape and then order or make hundreds of tiles, ceramic or wooden, following these shapes. The symmetry of each shape helps a lot. Let us examine two such examples.

**Fig. 1. Stone pattern from Cairo Complex of al-Ghuri Mosque**

In this pattern, we deal with a limited number of uniform shapes: squares (2 sizes), halves of square (2 sizes), rhombi (1 size), halves of rhombi (2 sizes), and frames.

Each shape was made by hand, and a limited number of shapes made this work much easier. Note: Each shape in this design has one or two mirror lines.

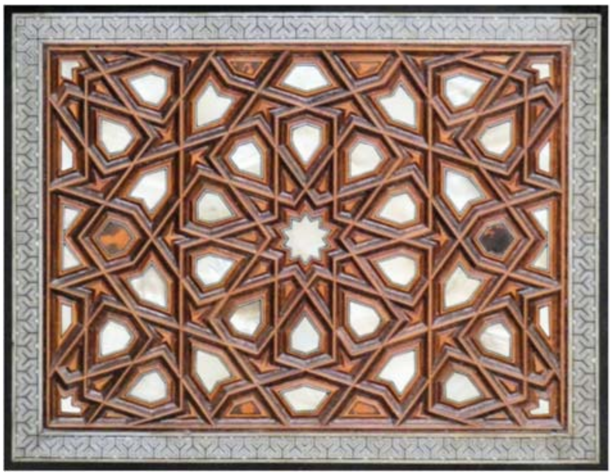


**Fig. 2. Fragment of doors from Sultan Ahmed tomb, Istanbul**

In this work, the woodworking craftsman used seven types of uniform elements. Exactly the same elements were used for other parts of the same doors. This way, the task of assembling such doors was much simplified but still very complex.

Again, each shape used in this design is symmetric with at least one mirror line. Only the central star has ten mirrors.

In this design, all lines cross without bending at crossing points.

The two examples are completely different. The first comes from the Byzantine mosaic tradition, where each shape is convex and a simple symmetric polygon. In such designs, lines are allowed to bend at crossing points. The second example is the gereh pattern following all gereh rules mentioned in my first paper in this series (see [7] ). Let me remind you briefly of these rules. We will need them for the following examples. In these rules, we deal with a tessellation of a larger polygon, referred to as a contour, and a pattern built on this tessellation. The first three rules describe the properties of the tessellation. The remaining four rules describe the relation of the pattern and its tessellation. Each of these rules can be ignored if this makes sense.

G1. Tessellation polygons are convex and symmetric.

G2. The tessellation is an edge-to-edge tessellation

G3. If a tessellation polygon is intersected by the edge of the contour, then it is always intersected along its symmetry line.

G4. If a single line of the pattern touches the edge of a tessellation tile or edge of the contour, then another line goes into the mirrored direction on the other side of the edge or of the contour.

G5. If two lines of the pattern meet on the edge of a tile or of the contour, then they continue without bending on the other side of the edge or contour. This simply means that the angles between lines of a pattern and the edge of the tessellation tile are equal.

G6. If two lines meet inside the tile, then a pattern following them may bend in this place.

G7. A line of the pattern may end only on the edge of the entire pattern.

## Where is the geometry?

While browsing the Internet or various publications, we often see drawings with strange features. Both examples in this section come from the Russian journal Молодой Ученый (Young Scientist), 2/2014, page 161. Each of these drawings is full of mistakes. Let us examine them.

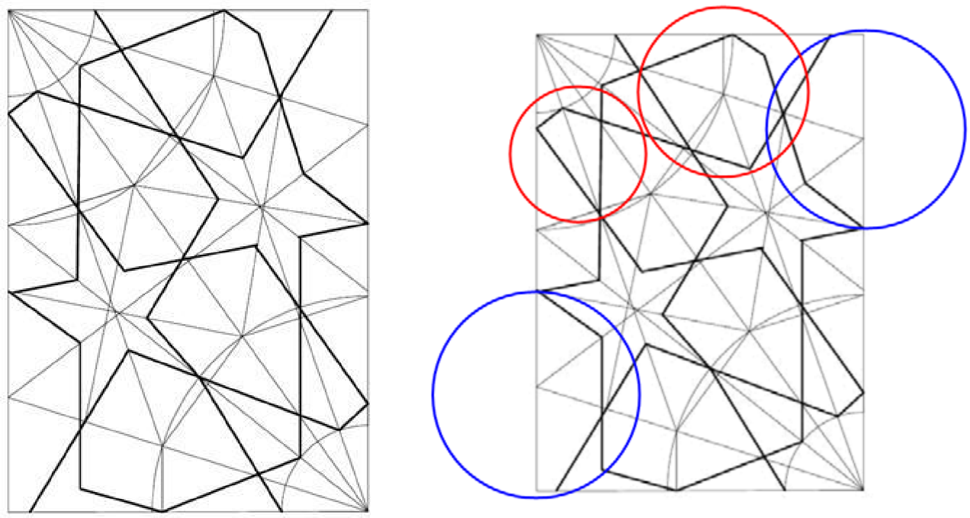
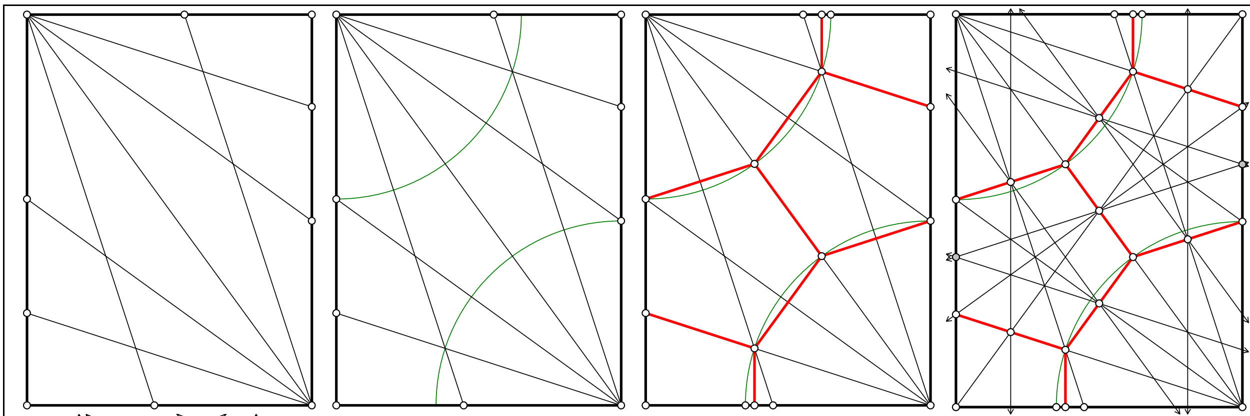


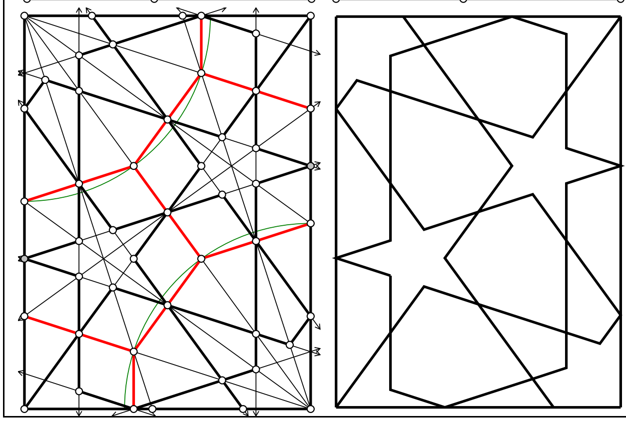
Fig. 3.

In the right drawing, the red circles show shapes that do not have any symmetry line. The fragment in the top-left corner should be part of a decagonal rosette, but the petals of the rosette will not be uniform. They are completely different.

The blue circles show strange and ugly shapes.

Let us see how this pattern should look in the correct design.





The final design is a construction of symmetric polygons, the two pentagonal stars are identical and each one has D5 symmetry. In top-left and bottom-right corners we have quarters of decagonal rosettes with identical symmetric petals.

This design is often seen geometric patterns in Iran, Uzbekistan and Turkey.

The second example from the same journal contains an octagonal design with similar mistakes, i.e., not symmetric and irregular shapes.

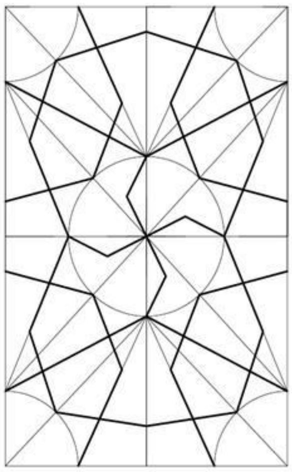


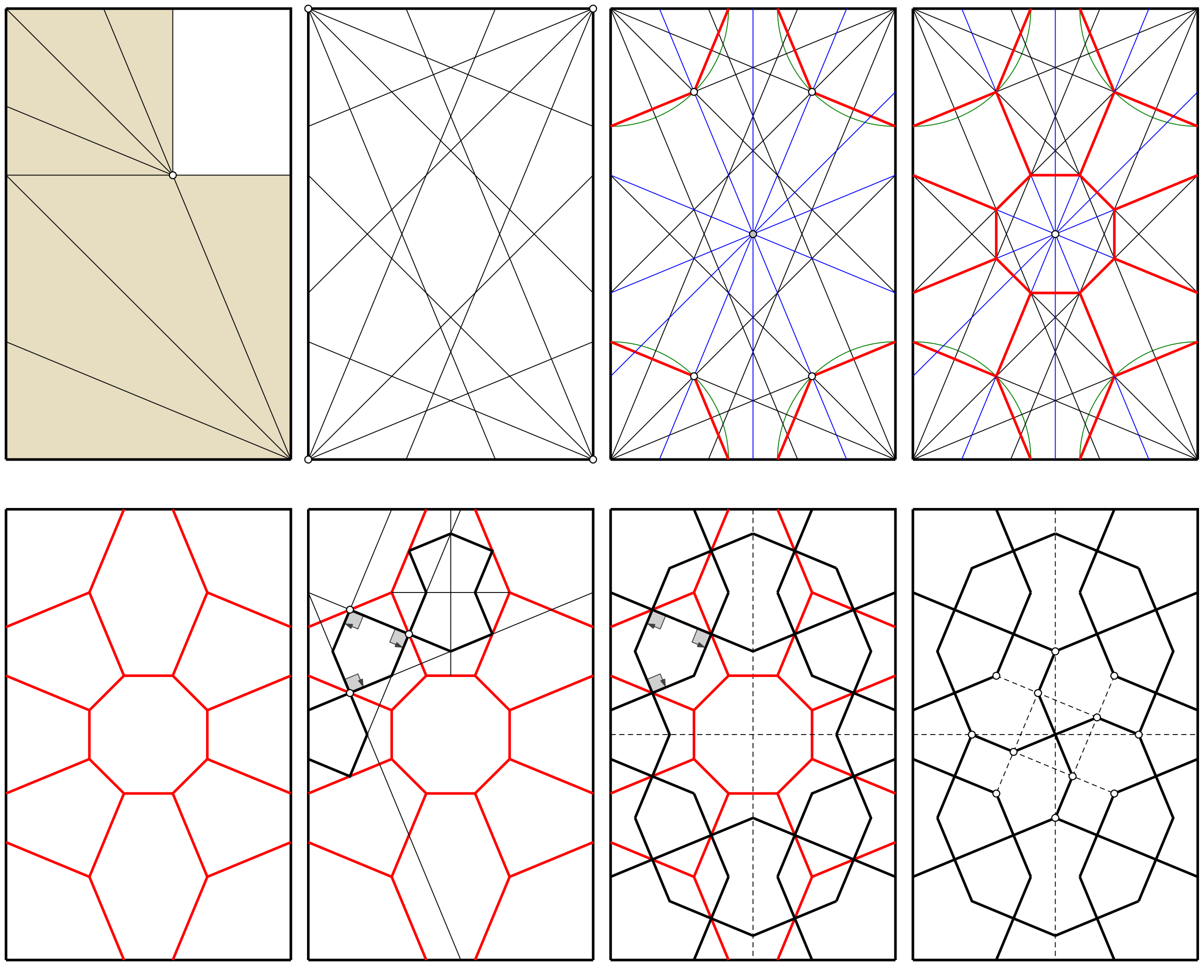
**Fig. 4. Another design from the Young Scientist Journal (2/2014)**

In this drawing, all shapes are irregular. Most of the shapes should have at least one mirror line. Some of them should have two mirrors. The stars in corners should use 90 degrees angles.

In the next series of drawings, we will show how to create the contour for a correct pattern, and then we will make a tessellation filling this contour. Finally, we will create a correct pattern.

Note – contour for this pattern is very specific, and it uses two stacked squares, one larger and one smaller. The whole pattern uses 90 degrees angles for most of its crossing points. The central part is added artificially to fill the large empty space in the middle.

Here are all steps of construction of the correct pattern.



Similar mistakes we can find in many other sources. We can find them also in professional designs and patterns on walls of monuments and mosques. In each case, authors are desperately trying to recreate a pattern forgetting completely about its geometry.

## Wrong assumptions

The design presented in this example shows what may happen if we draw a tessellation with the wrong starting assumption.

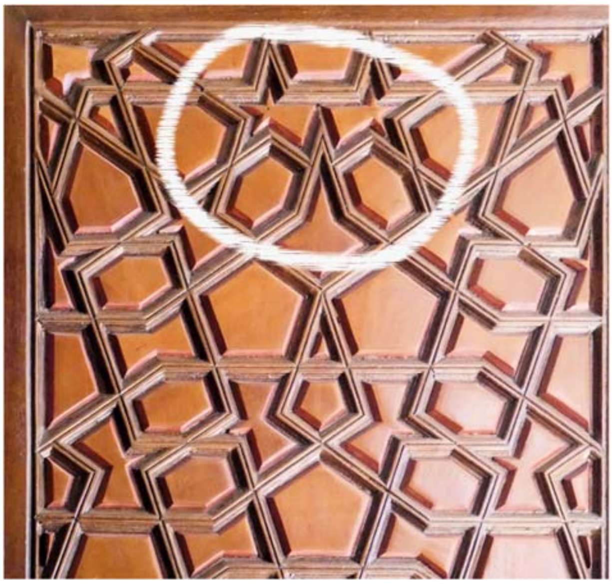


**Fig. 5. Fragment of window shutter from Mihrimah Sultan Mosque in Üsküdar, Istanbul**

Here we have two doubtful features. Lines of the pattern inside the marked area are connected strangely, and the shape above this connection looks wrong to our eyes. Otherwise, all remaining shapes look correct, and lines cross without bending.

In this work, like in all previously discussed examples, there is a limited number of uniform elements, and each has at least one mirror line.

The errors shown in the photo were unavoidable due to the geometry of this pattern. In the next series of drawings, we can see what happened here.

Now, let us see how the pattern from Mihrimah Sultan Mosque was designed.

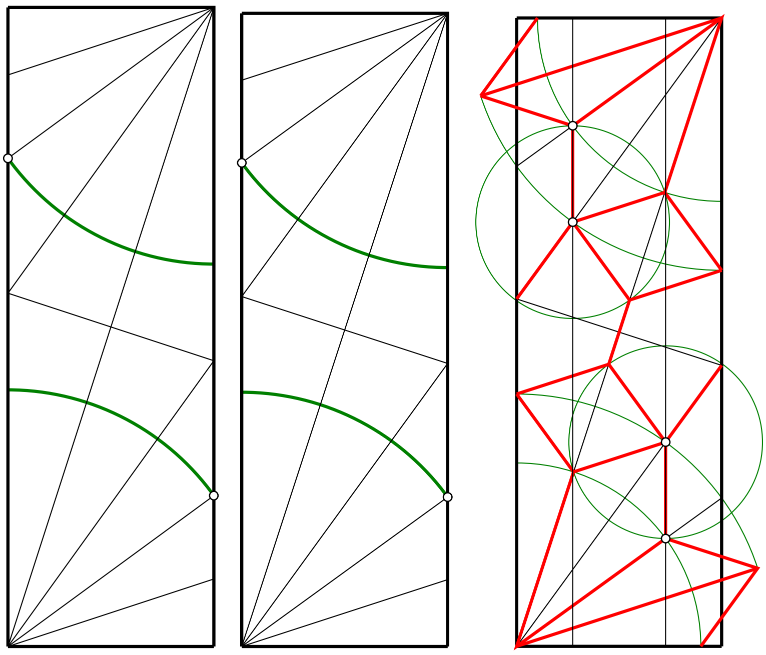
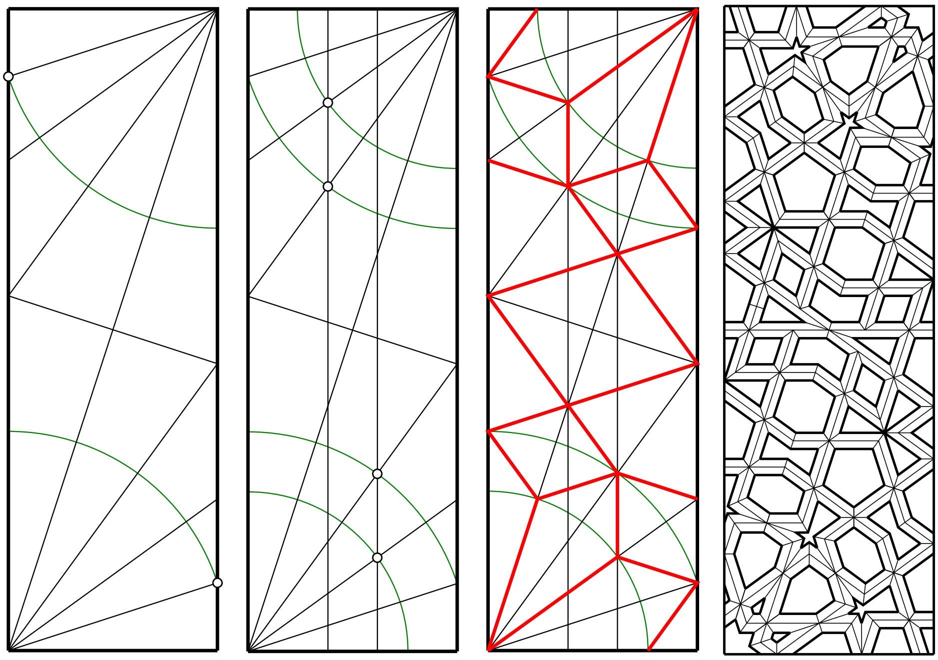


**Fig. 4. Construction of the tessellation (geometry) for the pattern from Mihrimah Sultan Mosque**

The points in the left drawing were selected as the radii for the two large arcs. This choice pushed parts of the tessellation outside of the contour edge.

The pattern from Mihrimah Sultan Mosque can be easily corrected. In the next drawings, we will show one of such constructions.



**Corrected pattern from Mihrimah Sultan Mosque**

As we can see here, by selecting different points for the arcs, we can push the long kites inside the contour and create a new tessellation. This way, we obtained a new pattern with a similar structure to the original design.

This pattern may have a few interesting variations, and each of them can replace the faulty design.

## Breaking rules

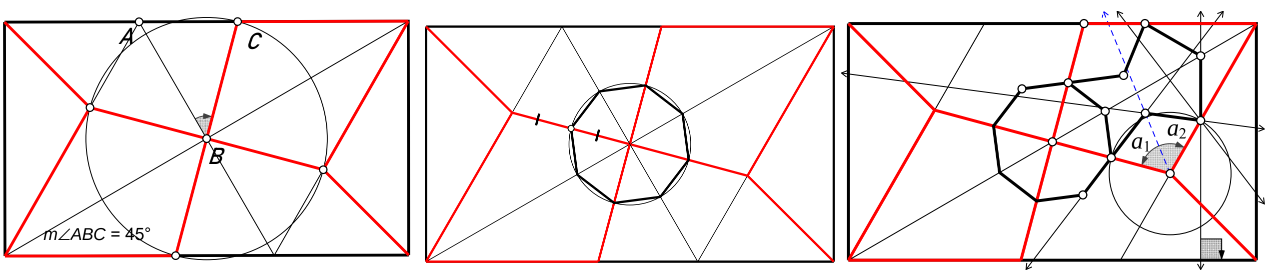
In old architecture all over Iran and Turkey, we can see designs breaking gereh rules mentioned at the end of the second section.

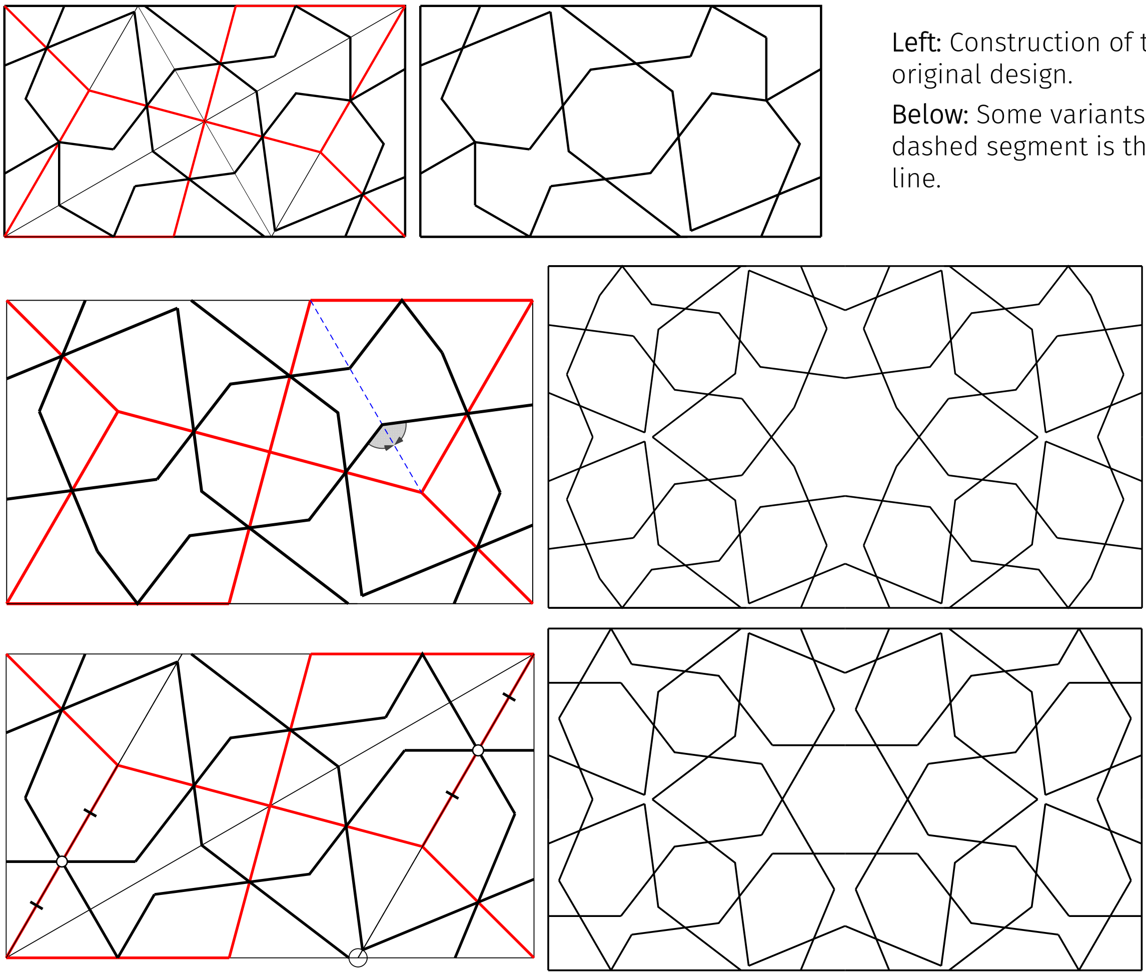
Our first example was taken from Green Mosque in Bursa.

**Fig. 6. Hexagonal example from Bursa**

In this example, there are some strange joints of pattern lines in each vertex of a hexagonal star. As we can see in a moment designer ignored here the gereh rule G5. But can we make this design better?

In the next drawings, we show step by step design of this pattern and some of its variants.





**Left:** Construction of the original design.

**Below:** Some variants. The dashed segment is the mirror line.

**Variant 1**

**Variant 2**

As we can see, the pattern from Bursa can be created in a few ways, but the question is, which of these versions looks better to our eyes? In the original pattern, the fish-like shape looks nice. In variant 1, they change to an ugly shape, and the star in the middle is too large, breaking the balance of the pattern.

Finally, variant 2, although it contains a large hexagon, still can be considered a reasonably good replacement for the original pattern.

We find a very similar problem in a drawing from one of the Bukhara scrolls.

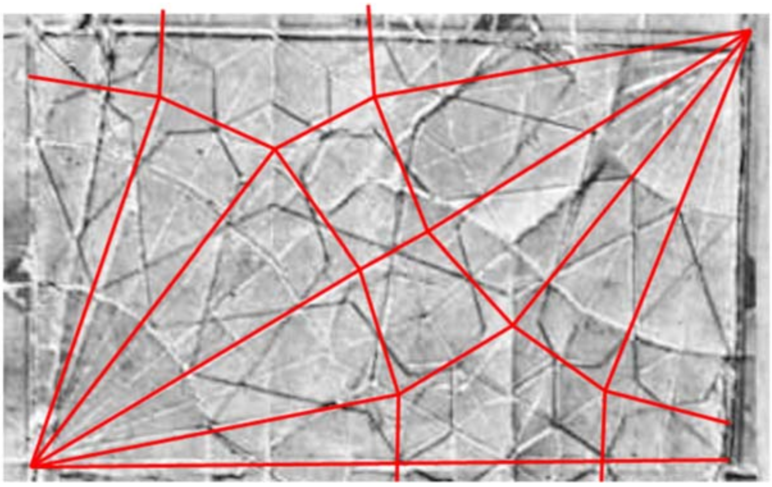
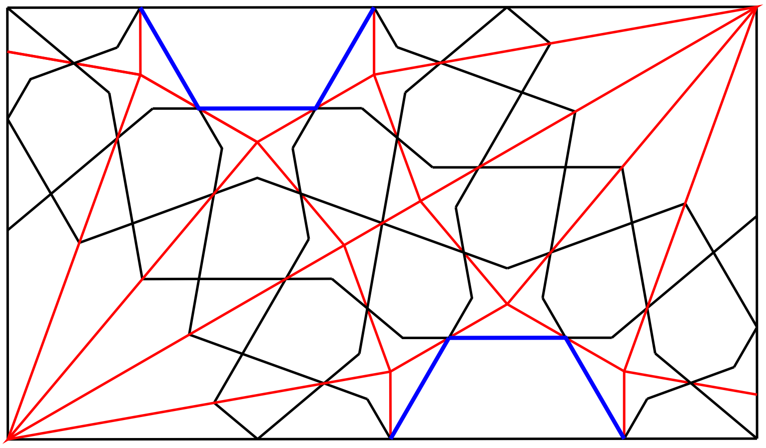


**Fig. 7. Pattern from Bukhara scrolls**

In this photo, we see one of many patterns from Bukhara scrolls. The red lines added by the author show outline of a possible tessellation for this pattern. The two hexagon halves contain a star that does not fit into this design. Its edges do not connect with the rest of the pattern in an expected way. Therefore, this pattern is often regarded as incorrect. But, the designer considered it acceptable.



The blue lines inside hexagons show how the pattern should be constructed if we strictly follow the gereh rules. We get large empty space that does not fit well with the pattern. Therefore adding there hexagonal stars was the best option.

This particular pattern we can find at the entrance to Khwaja Abdullah Ansari's tomb in

Herat, in Afghanistan.

There are many other patterns where bending lines in crossing points were used to create special effects or improve the visual quality of a pattern. Thus pattern designers often ignored the gereh rule G5 and other rules if they wanted to improve their works or make them more interesting.

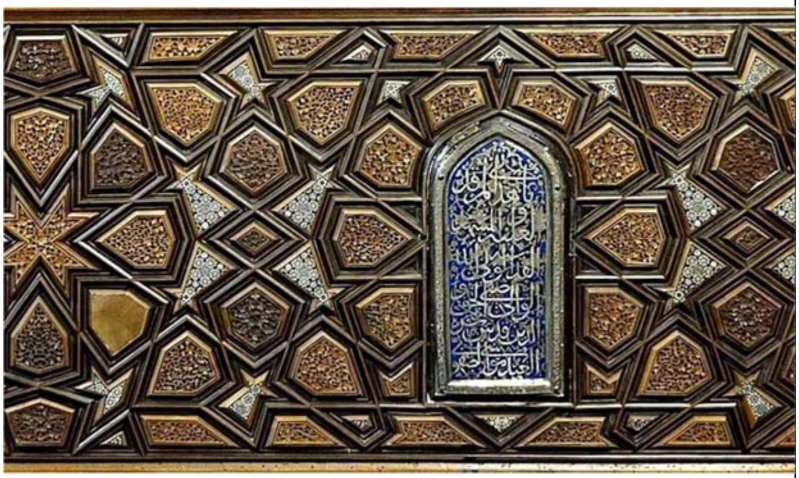
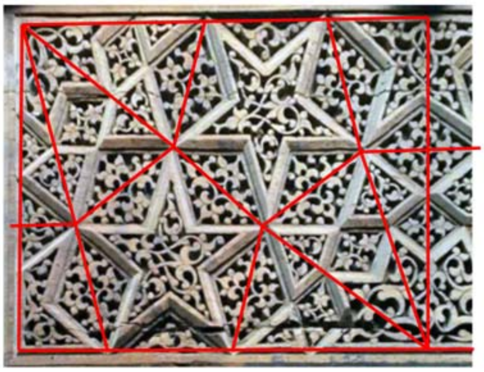


**Fig. 8.** Three patterns where gereh rules were ignored to get a more acceptable or special effects

The pattern to the left comes from Topkapi palace in Istanbul. The pattern shown above is a decoration from the tomb of Sheikh Safi-ad-din Ardabili, located in Ardabil, Iran.



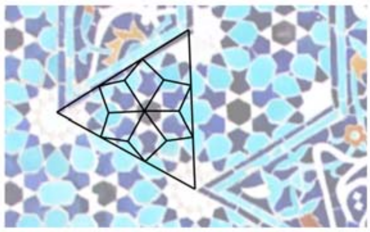
On the left, we have a fragment of doors from Birgi Aydınoglu Mehmet Bey Mosque, Odemis, Turkey. There is a large group of patterns using this approach. Although we have points where multiple segments are connected, no gereh rule was broken. The small stars in the top-left and bottom-right corners are not part of the pattern. These are extra decorations.

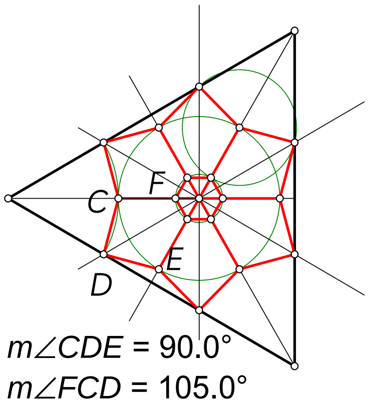
Persian designers frequently explored the concept of bending lines. Here is a magnificent wall mosaic from the Jameh mosque of Varzaneh, Iran.

**Fig. 9. Mosaic from Jameh mosque of Varzaneh**

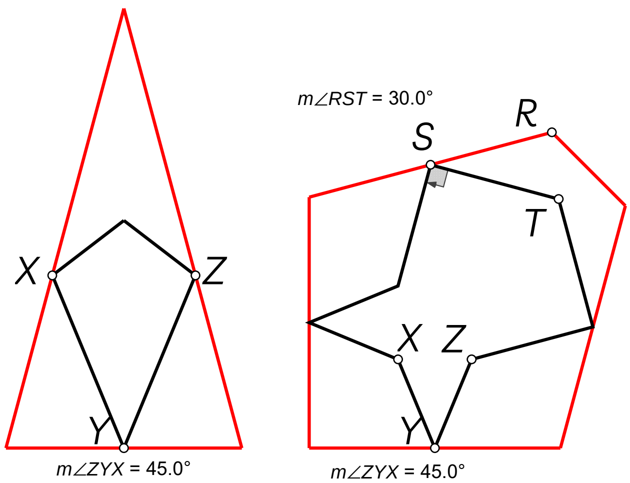
One can easily notice multiple places where lines of the pattern are bending. We can also notice the fish shape, which is one of the main elements of this mosaic.

The whole mosaic can be created on a triangular grid with a star of kites in the center of each triangle.



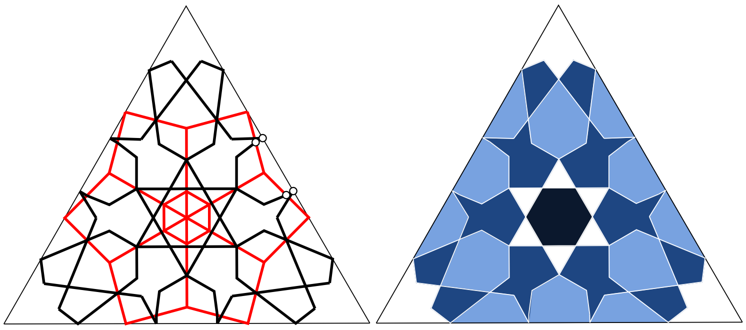



Construction of such a pattern can be done by designing a pattern for two elements only: the kite and one of the long triangles in the corners of the triangle.




**Basic shapes for mosaic from Jameh mosque of Varzaneh**

In these drawings, we show all angles used in this design. Note that the fish shape was created with angles of 90 and 45 degrees.



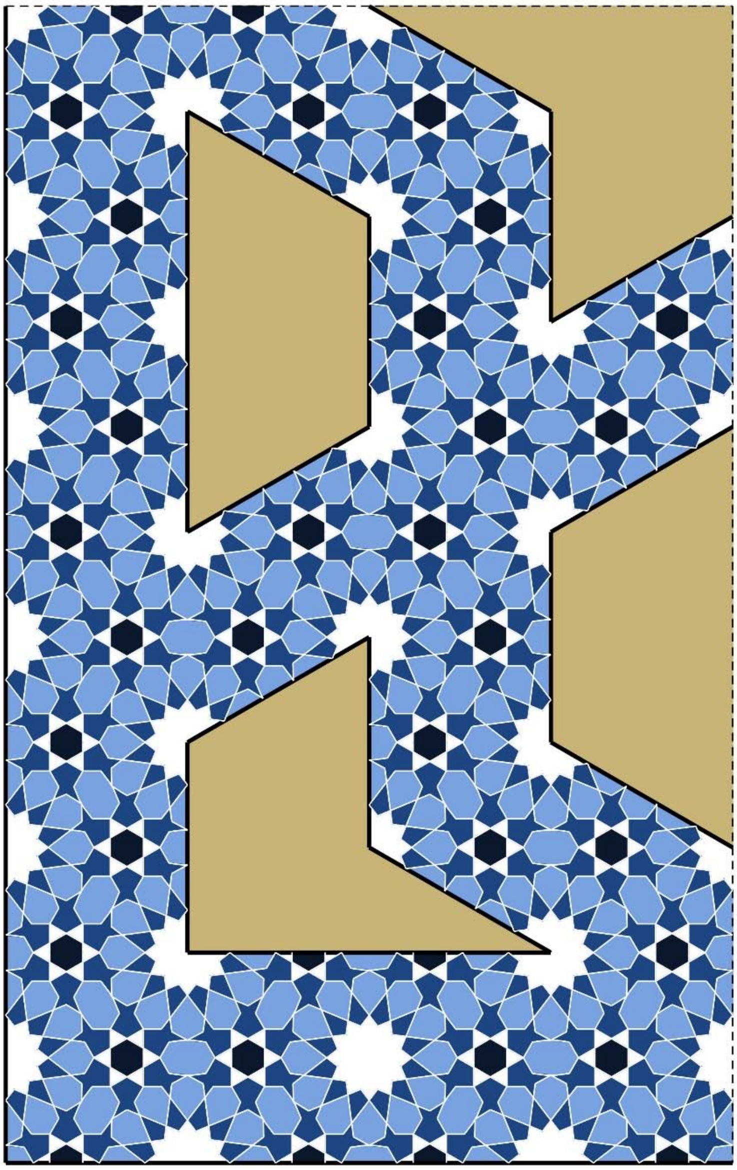
A complete pattern from the Jameh mosque of Varzaneh can be constructed by assembling the triangular tiles shown here.

### Complete construction of the pattern inside the triangle

We have to remove the red lines and place multiple copies of this module onto a triangular grid.

Note – the narrow red triangles near the center of each edge of the big triangle cannot be left empty. Thus we have an artificial fixing of the pattern. This part can be considered incorrect – three segments meet at one point.

This correction can be done in another way by distorting the fish's shape near this edge.



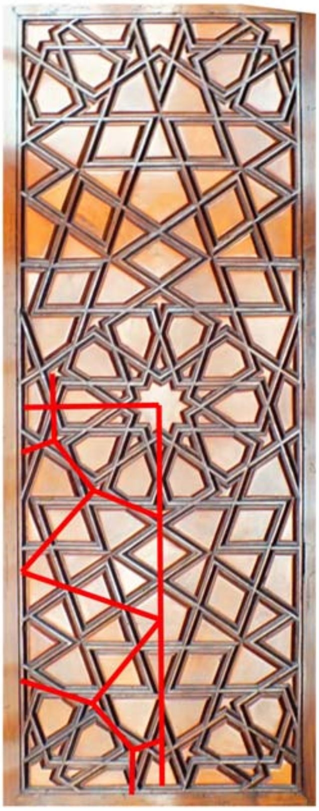
### Fragment of mosaic from Jameh mosque of Varzaneh

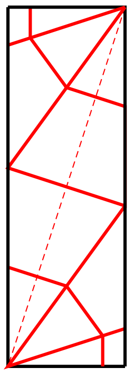
In this drawing, we show the bottom left quarter of the pattern from Varzaneh.

There is another way of creating a very similar pattern still using the same approach with hexagonal stars. A tessellation for this design uses three regular polygons only – dodecagons, hexagons, and squares. There will be no problems with narrow triangles in such a design, but dodecagonal rosettes will have some distortions, i.e., they will not have D12 symmetry.

## Standard versus non-standard

In each geometric pattern group, there are patterns with a particular type of tessellations that are considered a standard. But there are also some exceptional designs built on geometry with different proportions. These patterns are often considered wrong. In the drawing below, we show one of such patterns.





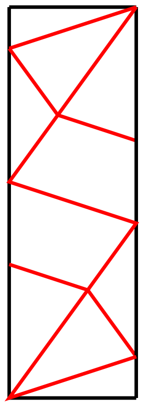
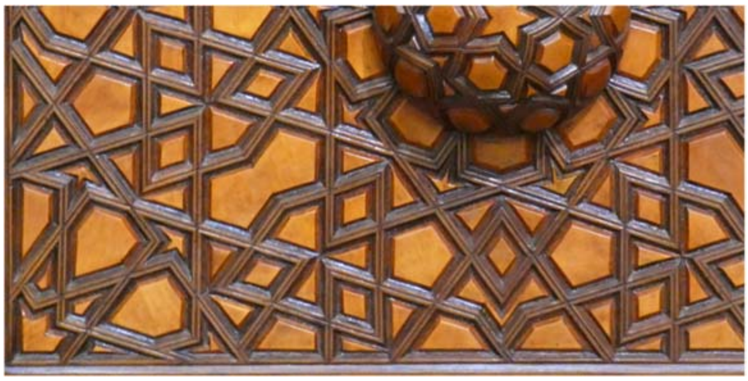


**Fig. 9. Window pattern from Rustem Pasha Mosque in Istanbul**

The pattern combines large shapes combined with relatively small petals of the central and corner rosettes. Its tessellation is shown in the left drawing.

Below-left we show tessellation that may be expected for such a design.

Below we show a pattern using standard proportions. Here the size of petals is larger with respect to other elements, and rhombi are smaller. In decagonal designs from Cairo, we can find more patterns with variable sizes of elements. They still follow the gereh rules.



Although decagonal patterns built with non-standard proportions of elements are not frequently seen, we still have to recognize their specific features.

Between Maghreb designers, there is also a very strong attachment to some specific octagonal designs. For centuries they constructed geometric patterns with exact proportions of elements. Therefore any pattern that does not follow these proportions is almost automatically considered incorrect. Let us analyze one of such cases.

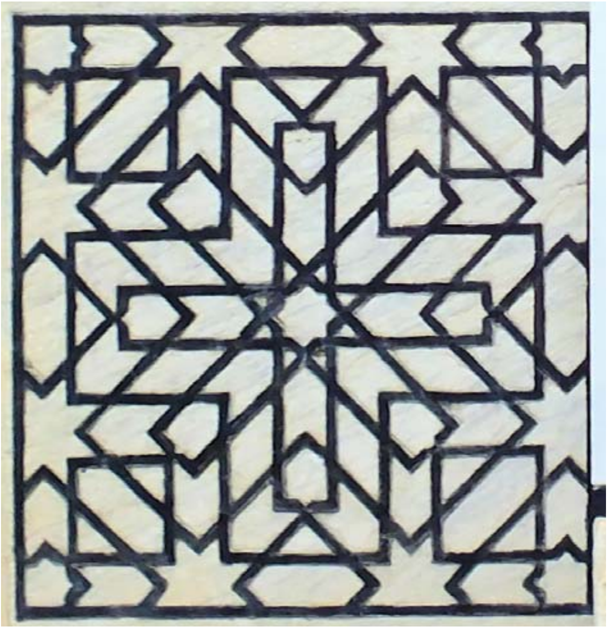


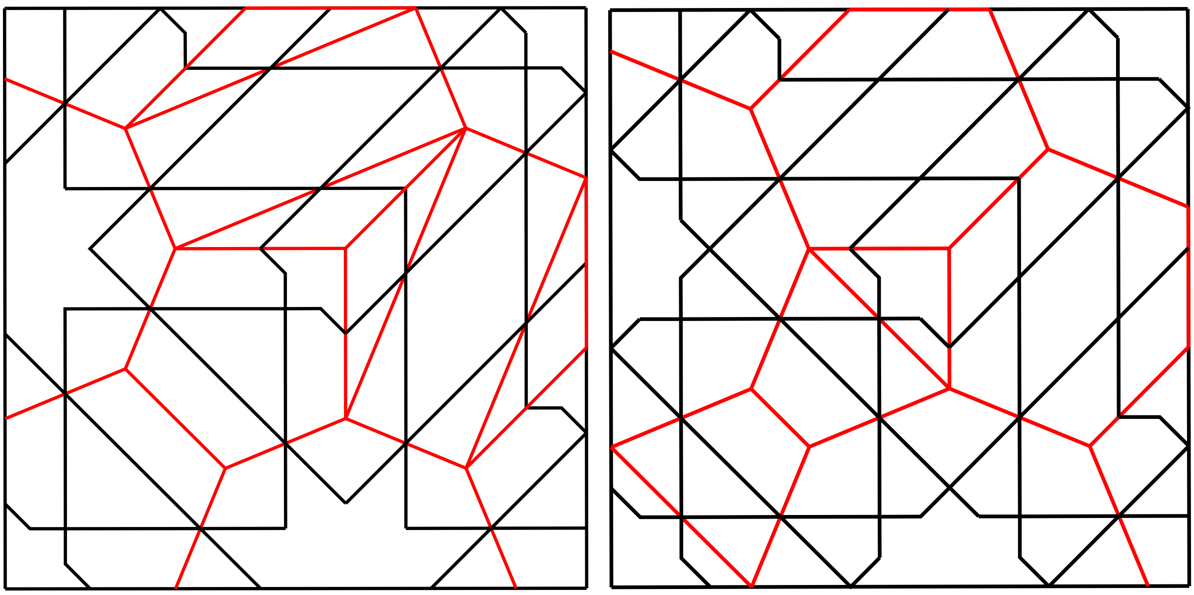
Fig. 10. Two patterns from Magreb (above)

Left – geometric pattern from the wall in Ez Zaytuna mosque in Tunis.

Right - one of many patterns in Saadian Tombs, Marrakesh.

Both patterns look very similar, but the left one is often considered incorrect. Only after a very detailed examination will we notice some minor differences. In the Ez Zaituna patterns, some shapes are narrower and longer than appropriate shapes in the pattern from Marrakesh. However, each of these patterns follows the same gereh rules. How do we get these differences? They use different tessellations.

One can also notice that we can construct a few more tessellations allowing us to produce a few more patterns similar to these on our photographs.



Left: Ez Zaytuna pattern and its tessellation,

Right: Pattern from Saadian Tombs

## Tiles and patterns

There is a popular belief that many decagonal geometric patterns were "constructed" by assembling decorated tiles, so-called "girih tiles." Assembling patterns the way we used for assembling puzzles was never used by medieval designers. L.I. Rempel, in his book, complained that young designers, after discovering tessellations for decagonal patterns, started developing patterns by simply manipulating decorated tiles and they forgot or never learned the traditional methods of pattern design. We often see the results of such an approach in patterns created in the last two centuries. Let us examine one such pattern probably created by assembling decorated tiles.

Camlica Mosque is one of the recently built mosques in Turkey, and it is considered a major mosque in Istanbul. In this mosque, we can find a few interesting geometric patterns. Some of them are modifications of existing Ottoman designs. One of them contains a few errors showing that the creator of this pattern could not complete it according to gereh rules. However, a good knowledge of the properties of decagonal polygons' may help us correct the pattern.

In the next photo and further drawings, we show the original pattern and how one could correct it using the properties of polygons.

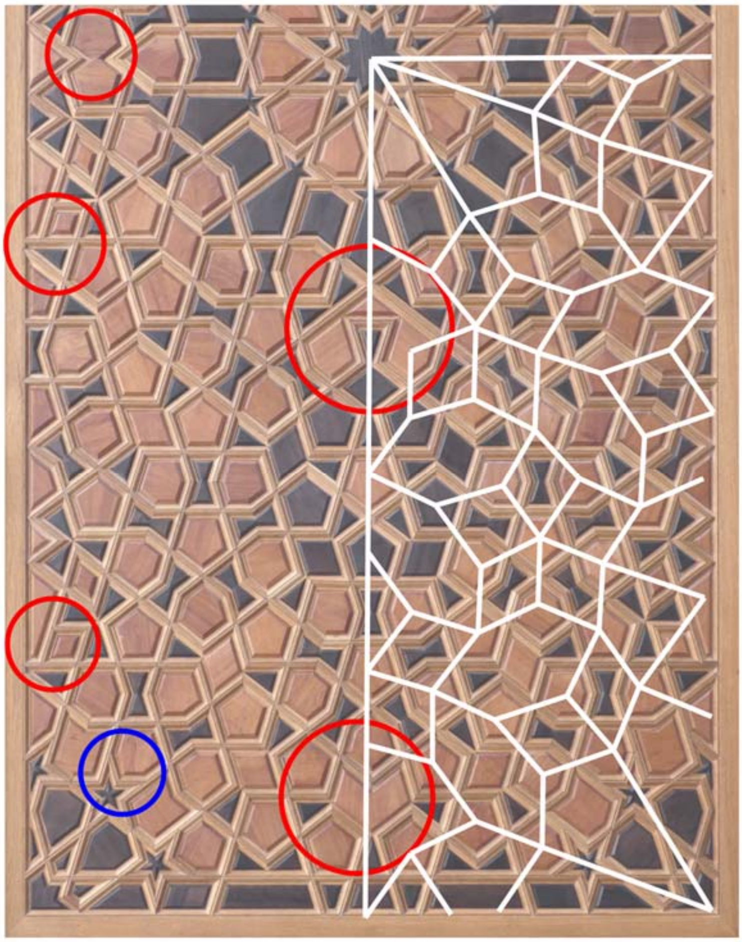
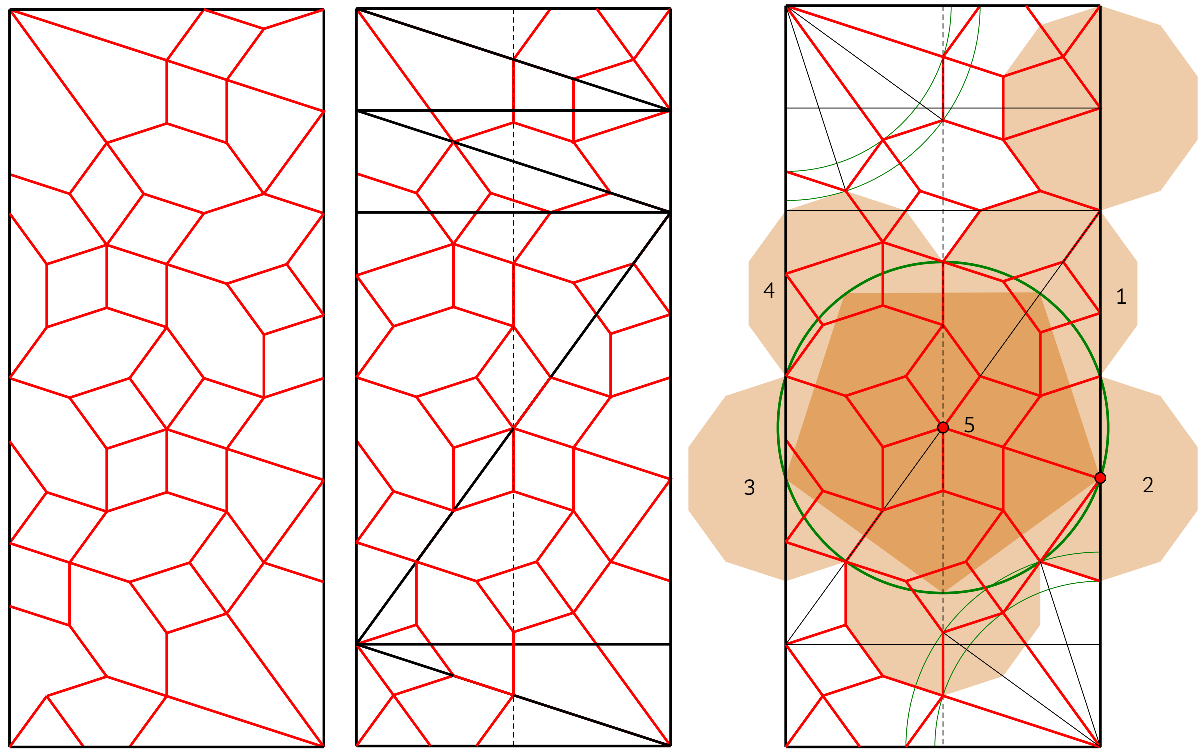


Fig. 11. One of Camlica kundekari doors

The left side of the photo shows places where the design is faulty (red circles). The blue circle shows a feature that should be considered an extra decoration. We do not consider it an error.

The three circles near the left edge point out inaccuracies that can be easily corrected. The two circles in the middle mark the most significant errors in this pattern.

The right side of the photo shows a possible tessellation for this pattern. Note what strange polygons we get in places where are errors.




The left drawing shows the original tessellation of the Camlica pattern.

The middle drawing shows a corrected tessellation.

The right drawing shows how the corrected tessellation was obtained: create the four stacked contours. Draw the pentagon using the red points and tangent decagons on their edges. Draw long kites in the top-left and bottom-right corners.

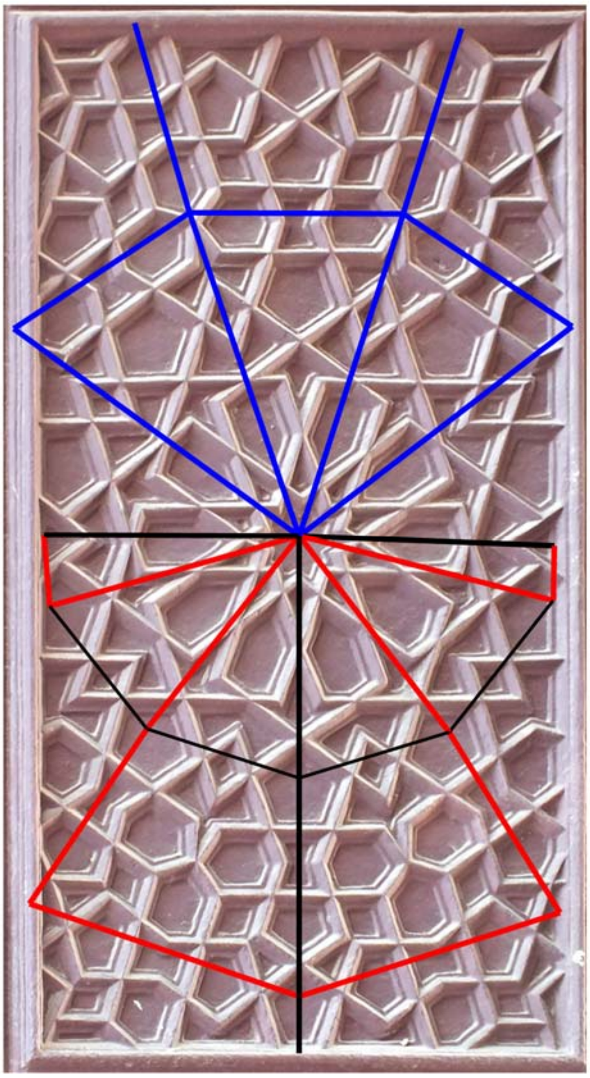
It is important to notice that this pattern may have a few correct variants. Everything depends on how we fill with pattern decagons and the space between them. Decagons 1, 2, and 3 can be inflated in a few different ways. The large star, nr. 5, can also be redrawn using different polygons. Finally, the space above decagons 1 and 4 may have at least one more polygonal representation.

## One of the most controversial patterns from Istanbul

Thousands of visitors to the Topkapi Palace pass the main entrance gate[3] without even noticing a very interesting pattern decorating its inner sides. We do not know much about the history of these doors and their pattern on them. According to some contemporary documents, this is the original pattern made during the creation of the Imperial Gate. However, this information is not confirmed by any reliable documents or drawings. It is not a new design, and judging by the view of the pattern and its creation technique, we can assume that it is an original work or a later replica. This pattern is unique and does not exist in any other place in the Ottoman Empire.

As we will see later, the pattern on the doors of the Imperial Gate is a small fragment of a larger design cut down to fit the provided space. Thus its edges do not follow the symmetry lines of shapes used. This is the feature that we sometimes see in early Ottoman designs.

[3] *The Imperial Gate of the Topkapi Palace in Turkish is called Bab-i Humayun. This is the outer most gate in the Topkapi Palace, beside the Sultan Ahmed III Fountain. This massive gate opens into the First Courtyard. The Gate was built in 1478 during the Sultan Mehmed II (the Conqueror) era, it is now covered in 19th-century marble. (text from https://www.islamichistoryandtravel.com/topkapi-palace-istanbul-first-courtyard/)*

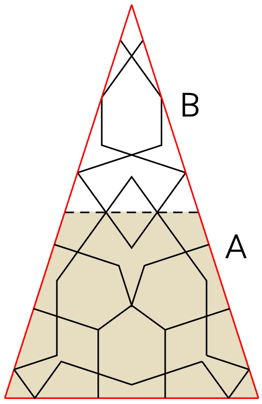


### Fig. 12. Fragment of doors in Imperial Gate

By analyzing the borders of this artwork, we can easily notice some unusual shapes. This suggests that we deal here with a fragment of a larger design.

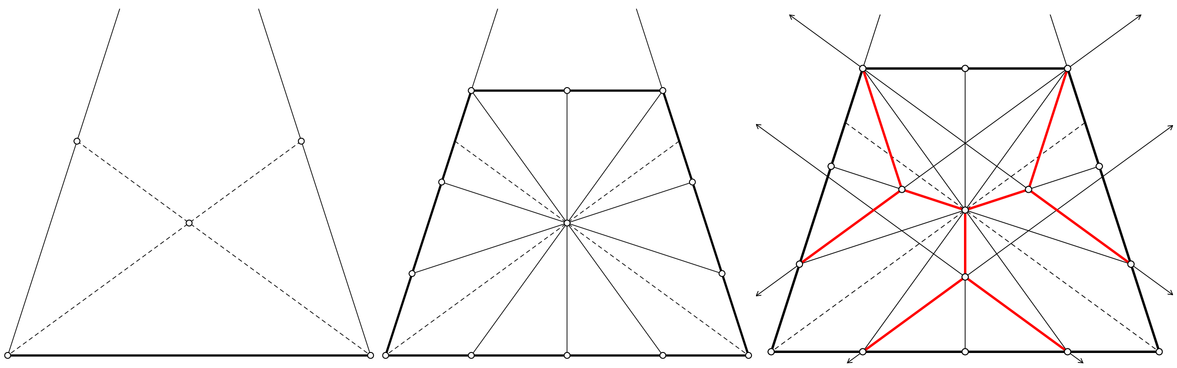
The geometric structure of the pattern can be presented using at least two types of structures – red polygons and blue polygons.

The blue polygons are regular pentagons, symmetric trapeziums with three edges equal, and a decagon. This is the usual approach that we often see in decagonal designs.



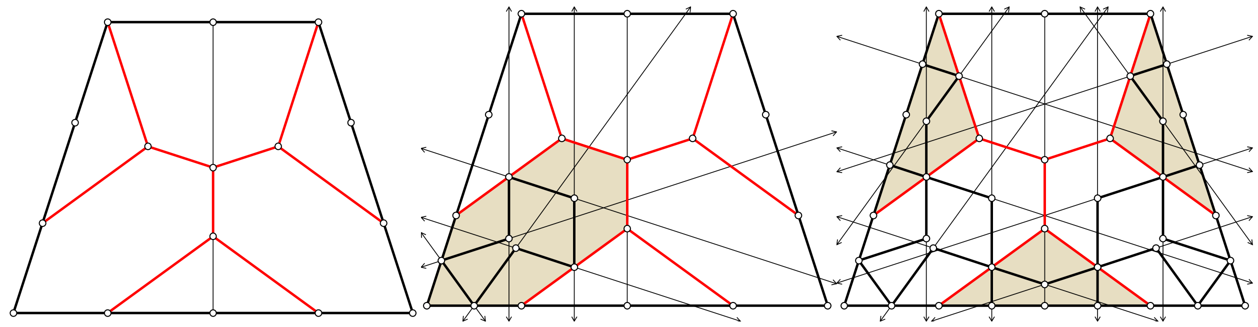

The red triangles together form a decagon, and each of them can be split into two shapes: a triangle and a trapezium with unusual proportions. The triangle part is a typical Ottoman style design often seen in Ottoman kundekari artworks. The drawing below shows both parts combined. The white part is the mentioned Ottoman style fragment. The shaded part is a specific motif occurring only in the Imperial Gate pattern. Note also – the petals of the rosette in the photo are elongated. This is also an unusual feature of this design. In later designs, the petals of a decagonal rosette have different proportions.

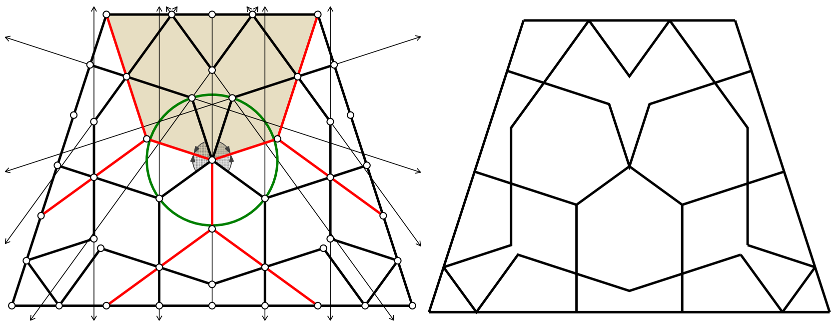
In further geometric constructions, we will follow the red polygons approach. This way, we can explain better the Ottoman rosette construction. We will start by constructing a pattern for the shaded trapezium, extending it toward the rosette part.



### Construction of the tessellation for the trapezium

All steps in this construction are obvious. It is also possible to design it with smaller triangles on the sides and wider hexagons in the bottom corners. This way, we may get a very similar pattern but with different proportions of shapes.

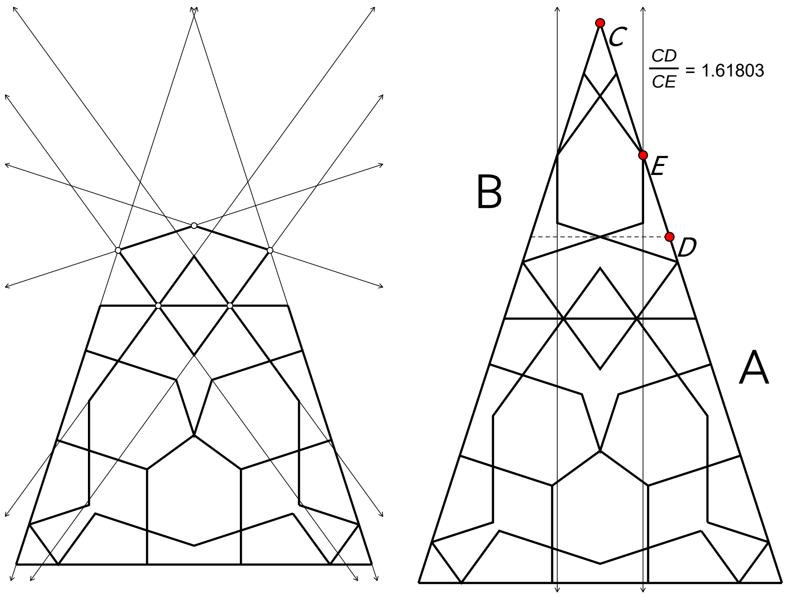




### Steps for constructing pattern inside the trapezium

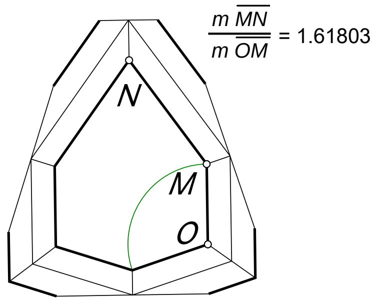
The circle in the middle shows how the central petals were constructed. This place is often considered controversial. Note – all angles between red lines and segments on both sides are equal (compare rule G5).

Now we can build the Ottoman rosette with narrow petals. In many other Ottoman designs, this rosette was created with wider petals with a ratio L/s (long edges divided by short edges) equal to the golden ratio.

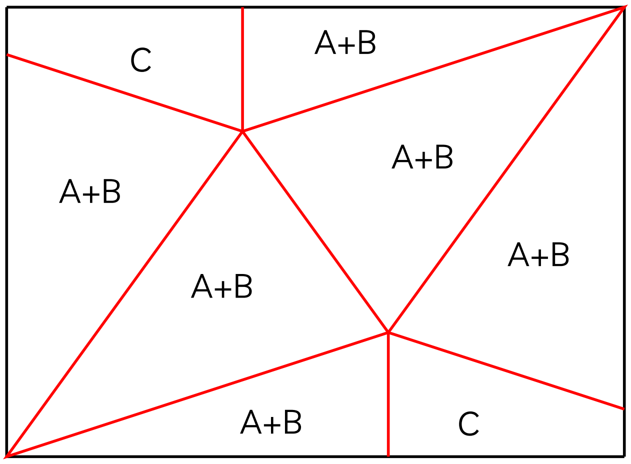



### Rosette construction

Note how the width of the petal was established. Point E was selected using the golden section of the segment CD. But, point C can be selected in any other way, and the petal can be wider or narrower. Below I show the more often used rosette petal proportions.



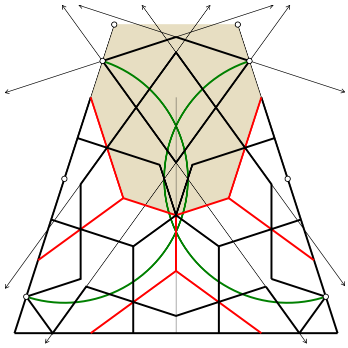

Finally, we have to construct an element missing in the photo of the Imperial gate pattern, but it is necessary to construct a rectangular design using this approach.

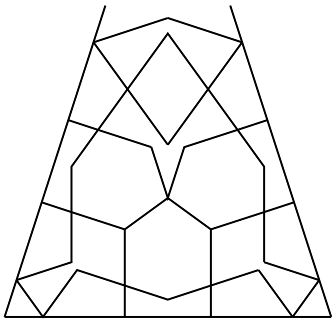



## Elements of tessellation for the pattern from Imperial gate

Triangles A+B are places where we will fit the pattern obtained in the previous drawing. Here A and B mean both parts of the large triangle.

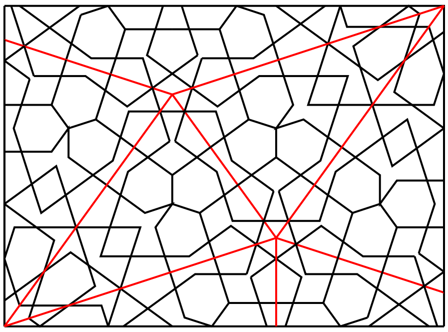
We still need to construct a pattern for trapezium C. This trapezium has all long edges of the same length. The pattern near each long edge must not conflict with the pattern at the bottom of the triangle A+B.





## Construction of element C

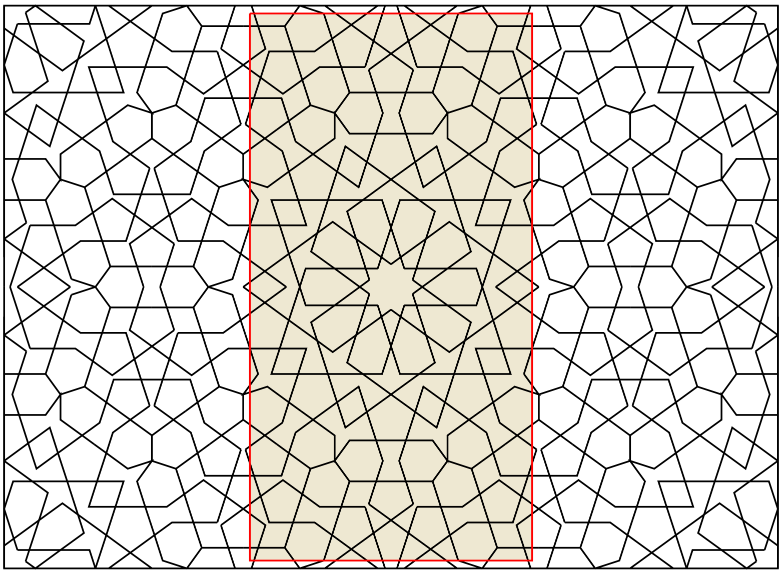
Element C is an extended version of the trapezium from A+B. The two arcs were used to make the pattern match the bottom part of the pattern in the A+B triangle.



## Final construction of the pattern from Imperial doors

Left – a template for a large pattern using elements A+B and C.

Bottom – a large pattern created from this template. The shaded area shows how the pattern from the Imperial gate was cut out of a larger design.



## Summary

There are many other geometric patterns with features not always following the gereh rules. It seems that the medieval designers did not always care about fitting a pattern correctly in the provided space. Thus we often see errors near the edge of a pattern. The rule G5 was often ignored for the sake of getting symmetric shapes of the pattern. In general, for a craftsmen pattern is a combination of polygons not a network of lines. Thus the symmetry of shapes was more important than forking or bending pattern lines.

**FINAL COMMENTS:** For this paper, there are no supplementary materials. The projects presented here should be created by interested readers using paper, pencil, compasses, and rulers. One can also use any software for high school geometry. This way, readers can better understand the presented here concepts and start experimenting on their own with geometric patterns. All this is along with the famous saying 'no pain, no gain.' All drawings and geometric constructions in this paper were created using the free version of Geometer's Sketchpad.

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(*ver. 4; updated on April 19, 2022*)